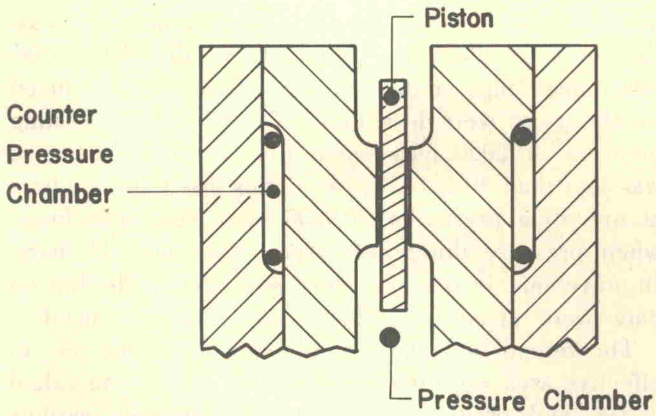

FIGURE 2. Geometry of controlled-clearance gage showing counter pressure chamber.

cylinder in order to assure the proper analysis of the piston distortion. This distortion generally is much smaller than the distortion of the cylinder, and thus the analysis is more reliable. Third, in the evaluation of elastic distortion in the simple free-piston system, Dadson (1955, 1958) proposed the use of the "similarity" principle in which two gages are so constructed that intercomparison between them will yield distortion information of both based on rather broad assumptions that the elastic deformation is similar in the two systems rather than using detailed elastic theory on either alone. In a second approach of this type called the "flow" method, Dadson has also shown consistency with the similarity method using the general theory of viscous flow without recourse to detailed viscosity values or analysis but by requiring consistency between related gages of specific dimensions.

All three of the above approaches have improved our understanding of the inherent deformation problem in a piston-cylinder gage and have independently demonstrated improved reliability and precision. Furthermore, the intercomparison of the three approaches afforded by the measurement of the freezing point pressure of mercury at 0 °C and approximately 7.570 kbar as discussed elsewhere in this report allows critical evaluation of the relative validity of the approaches. Since our current evaluation of the primary pressure scale is based to a large measure on these three approaches, each will be discussed in order.

a. Theoretical, Followed by Gage Intercomparison

The rigorous solution of infinitely long cylinders and pistons is straightforward, and the solutions are known as the Lamè equations. As a first approximation, Johnson and Newhall (1953) made some simple calculations based on an elementary theoretical treatment in which they assumed that the drop in pressure in the crevice along the length of the piston takes place over a small interval and is uniform below and above this point. The Lamè equations were assumed to apply to the long cylindrical sections above and below. Using some simple cases, they showed that the change in effective area varied linearly with the measured pressure in this approximation and that the pressure coefficient λ in the relationship $A_e=A_0\ (1+\lambda P)$ was of the order of 3×10^{-7} per bar for a simple free-piston gage made of steel. They illustrated uncertainties in λ of the order of 100 percent depending upon where the sharp pressure drop occurred along the piston length, where the piston was located in its travel, and also upon detailed construction of the piston-cylinder assembly. At ten kbar, this represents an uncertainty of the order of 0.3 percent. Johnson, et al. (1957) later reported a brief experimental comparison of a simple piston gage with a control gage assumed as the standard. The distortion of the simple gage even at pressures below 300 bars was shown to be linear, and the measured distortion coefficient λ indicated a pressure gradient near the upper portion of the piston. Zhokhovskii (1959a, 1959b, 1960, 1964) has carried out a rather straightforward but thorough analysis based on the assumption that the change in radius a of the piston and radius b of the cylinder at a general position x, along the length of the piston could be evaluated using the Lamè equations in terms of the pressure in the crevice at the point x. Since the Lamè equations apply rigorously only to long rods and hollow cylinders where the pressures are uniform along the length, one would expect serious discrepancies if the pressure variation along the piston length were abrupt, but the assumption would be more satisfactory if the pressure gradients were small and appeared over a large portion of the crevice length. Zhokhovskii's analysis makes no other assumption as to the functional variation along the length of the crevice but does assume perfectly cylindrical geometry and that no other deformation takes place except that described above by the Lamè equation.

Zhokhovskii's analysis was carried out for both simple piston gages and gages using cylinders with counter pressure. The latter is an extension of Bridgman's re-entrant cylinder system in which the chamber pressure extends completely along the working length of the piston. This system should not be confused with the controlled-clearance gage designed by Johnson and Newhall for which the geometry is significantly different and the counter pressure is independently adjustable from the measured pressure. In all cases studied by Zhokhovskii, the effective area was shown to change proportionally to the pressure being measured. The proportional relationship can be written alternatively in terms of the change in area ΔA or in terms of the pressure difference ΔP between the true pressure and the calculated pressure assuming no change in area. Thus

$$A_e=A_0\ (1-\lambda P),\ \frac{\Delta A}{A_0}=-\lambda P,\ \text{or}\ \Delta P=-\lambda P^2. \quad (1)$$

Since λ is small $\Delta P \ll P$ and the three equations are equivalent within the linearity approximation.

Zhokhovskii's theoretical results can be summarized in one equation which gives the value of the constant λ in terms of Young's moduli E and E' for the cylinder and piston respectively, the Poisson ratios σ and σ' for the cylinder and piston, the radius b of the piston, and the internal and external radii a and R of the cylinder:

$$\lambda=\frac{(3\sigma'-1)}{E'}+\frac{1}{b}\,(k-k_1) \tag{2}$$

where

$$k=\frac{a}{2E}\left[\frac{R^2+a^2}{R^2-a^2}+\sigma\right]+\frac{b}{2E'}\,(1-\sigma') \tag{3}$$

and k_1 has a different value for a piston with a regular cylinder and a piston with a cylinder with counter pressure. For a regular cylinder

$$k_1=\frac{b\sigma'}{E'} \tag{4}$$

and for a cylinder with counter pressure

$$k_1=\frac{a}{E}\left[\frac{2R^2}{R^2-a^2}-\sigma\right]+\frac{b}{E'}\,\sigma'. \tag{5}$$

In the case of the cylinder with counter pressure, the use of the Lamè equations for the cylinder appears rather questionable since near the upper end of the cylinder the internal pressure approaches atmospheric pressure but the counter pressure is still the pressure being measured. Since in Zhokhovskii's approach, the pressure used in the Lamè equations is the pressure in the crevice, an obvious contradiction to physical reality occurs.

Zhokhovskii and coworkers have published several experimental papers—Zhokhovskii and Bakhvalova (1961), Bakhvalova (1964), Zhokhovskii (1958)—showing the result of intercomparisons of gages with a variety of dimensions and material of construction, some with cylinders using counter pressure and some with regular cylinders. Comparisons were made using a differential-resistance high-pressure gage (Zhokhovskii and Bakhvalova, 1960), consisting of two manganin coils used as pressure transducers. Each resistance coil was placed in a separate pressure chamber with a valve connecting the two chambers, and the coils were electrically connected into parallel arms of an equal-arm Wheatstone bridge. The bridge was balanced at high pressures when the two pressure chambers were communicating with each other. The valve between the two was then closed, a free-piston gage was attached to each chamber separately, and the gages were caused to float simultaneously. Since the chambers were not in communication, the unbalanced Wheatstone bridge yields the pressure difference existing between the two free-piston gages. Repeated measurements to check the reliability of the differential-resistance gage indicated uncertainties introduced by the gage were less than 0.5 bar at an operating pressure of 2000 bars when the pressure difference was less than 40 bars. Uncertainties less than two bars at operating pressures to 9000 bars were introduced when pressure differences were less than 200 bars. Improvement in this precision was later made, but no data were given to evaluate magnitudes involved.

Theoretical calculations of expected changes in effective area were made on each of several individual gages, and predicted discrepancies between various sets of two gages were calculated. The experimental intercomparisons of the regular piston-cylinder assemblies were made using the technique described above. At ten kbar agreement of the experimental discrepancies with the predicted discrepancies was 0–5 bar. Correspondingly better agreement was obtained at lower pressures. In general, agreement for systems with counter pressure was approximately the same. Nevertheless, disagreement between theory and experiment as high as 15 bar was observed for one system using a cylinder with counter pressure. Further experimental work on the systems with counter pressure, including bell mouthing the cylinder at the upper end, indicated that for cylinders with counter pressure the crevice near the top of the piston becomes too restricted. Zhokhovskii also designed and constructed a gage which, based on the same theoretical approximation, would have an effective area that would not change with pressure. Experimental intercomparisons, however, indicated uncertainties of the same order as given above.

From a consideration of these results, one concludes that a primary pressure scale based on the equation proposed by Zhokhovskii accompanied by appropriate testing to assure the absence of taper, noncylindrical geometry, or unsuitable clearance could not possibly yield a scale more accurate and reliable than five bars at ten kbar and in general would be less reliable.

b. Controlled-Clearance Gage

The controlled-clearance gage developed in 1953 by Johnson and Newhall and illustrated in figure 2 eliminates immediately the problem of excessive leakage at the higher pressures. The increased complexity is offset by the additional versatility available in varying the jacket pressure, thus permitting measurements to be made at an optimum piston fall rate at all times. An inherent improvement in sensitivity and convenience thus results. Nevertheless, different sized piston-cylinder assemblies are desirable to give higher sensitivity at lower pressures, although a single system can be used from zero to the maximum pressure. In such a gage, the initial clearance is less critical for